Designing Tasks to Promote and Assess Mathematical Transfer in Primary School Children

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This study aims to design learning situations and tasks that promote and assess the capacity of primary school children to transfer mathematical knowledge to new contexts. We discuss previous studies investigating mathematical transfer, and particularly the strengths and limitations of tasks used to assess transfer in these studies. We describe some pilot tasks that were used with upper primary children and provide some responses to teacher prompts. We describe some design principles for the construction of tasks and an associated categorisation of prompts that might be used as the basis of further research into mathematical transfer.

The *Australian Curriculum: Mathematics* states that "[m]athematics is composed of multiple but interrelated and interdependent concepts and systems which students apply beyond the mathematics classroom (Australian Curriculum and Assessment Reporting Authority [ACARA], 2013, Rationale). However many of the typical so-called real world applications of mathematics amount to little more than pseudo problems, posed in an artificially constructed context, in which it is obvious which particular mathematical concepts are intended to be used (Beswick, 2011). Such situations require, at best, near transfer (Verschaffel & De Corte, 1997), in that the mathematical knowledge accessed in solving the problem is closely related to the mathematics recently learned. However, as the *Australian Curriculum: Mathematics* clearly states, a goal of school mathematics learning is to enable students to respond to both "familiar and unfamiliar situations". That is, a goal of school mathematics education is to promote *far* transfer (Australian Curriculum and Assessment Reporting Authority [ACARA], 2013, Rationale), in that the mathematics being accessed by students in solving a problem is not always immediately obvious, the context may be unfamiliar, and the mathematics may be something that has not been recently learned.

This paper reports on a study that aimed to create problems and associated interview prompts that could be used to assess upper primary students' capacity to transfer mathematical knowledge to unfamiliar situations where the mathematics required is not immediately obvious. We briefly review the literature related to transfer of mathematical knowledge which, despite many years of research in cognitive science, remains contested, and examine some attempts to assess that transfer. We then present some data gathered from video recordings of students to whom we posed some problems requiring transfer, and describe some design principles and prompt categories that we suggest might help in informing future attempts to develop assessments of far transfer.

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Literature review

Mathematical transfer

According to Brown and Campione (1984) the term transfer is used in "many different ways" (p.144), but is generally considered to be a process where students "leap across domains of knowledge" (p.144). These leaps can be near or far, conceptually, contextually or temporally. That is, they may use mathematics that is familiar or unfamiliar, be located in contexts that are familiar or unfamiliar, or access knowledge that has been gained recently or much longer ago.

Moreover the literature surrounding transfer is contested. Some researchers promote a relatively simplistic belief that transfer is a natural part of learning, while others suggest that all learning is contextual and hence transfer is unlikely. On the one hand proponents of transfer as natural suggest that deep conceptual understanding of mathematics is both necessary and sufficient to promote transfer. For example, Prawat (1989) states that mathematics and mathematical procedures that are "conceptually understood are much more likely to be accessed when needed" (p.10), while Hiebert and Lefevre (1986) claim that conceptual knowledge directly facilitates the transfer of this learning to other structurally similar mathematics tasks. On the other hand, those who hold a strongly contextual view of learning suggest that boundaries between mathematical knowledge, contexts and mathematical practices are seldom automatically crossed during mathematical learning (Boaler, 1993; Carreira, Evans, Lerman, & Morgan, 2002).

Assessing transfer

According to Ferrara, Brown and Campione (1986) transfer of learning is "rarely assessed" and is "relatively unexplored" in education research (p.1088). We argue that not much has changed until now as, with the exception of some somewhat contrived studies conducted with university psychology students in an artificially created environment (De Bock, Deprez, Van Dooren, Roelens, & Verschaffel, 2011; Kaminski, Sloutsky, & Heckler, 2008), very few recent studies have attempted to assess transfer of learning in mathematics. We could find none that attempted to assess far transfer, and none that were undertaken with primary aged students. We argue that new frameworks, design principles and tasks are needed to effectively assess transfer of learning in school mathematics, particularly across the diverse communities found in most primary schools.

Brown and Campione (1984) and Ferrara et al. (1986) used principles of dynamic assessment (Bransford & Schwartz, 1999; Carlson & Wiedl, 1992) to assess near transfer in school-aged children. The dynamic assessment approach sees assessment as a flexible contract between the student and the teacher, enacted through just-in-time prompts, rather than as a static test in which the teacher plays no role once the initial question is set. The number of scaffolded prompts required for students to successfully solve a new problem was then used as the measure of their capacity to transfer knowledge. The problems in these studies included letter series, in which children were taught how a pattern such as NGOHPIQJ was constructed and then asked to write down the next few terms of a pattern such as HQIRJS. This is a classic example of very near transfer, in that the assessment problem was identical in structure to the original problem such as that described above. The scaffolded prompts were given in a pre-set scripted sequence, independent of the individual child's responses, starting with general prompts and moving towards very specific, detailed prompts that the child could use to find the correct answer.

A slightly more problem-oriented approach was taken by Brown and Kane (1988) who assessed transfer in preschool children (3 to 5 year olds) by examining, through seven different experiments, whether children could "solve problems by analogy given repeated experience" (p.498). Children were required to use information in an example problem, which was modelled by the researcher, to solve a second. One example problem involved a garage mechanic stacking tyres to reach a high shelf. The assessment problem then involved a farmer who needed to stack his bales of hay on top of the tractor, but could not reach the top of the tractor. Children were encouraged to think out loud and explain their thought processes, while researchers asked open-ended questions to prompt their thinking such as "What do you mean?" or "Can you tell me a little more?" (p.503). Again, we argue that this assessment involves near transfer, in that the problems differ only in context, but not in kind.

Findings from the above studies suggest that "elaborations and explanations provided by the subject are more effective in promoting transfer than those provided by the experimenter" (Brown & Kane, 1988, p. 517), and that transfer is more likely when there is a focus on a broader set of understandings than when the focus is on a single rule. However as we have described above, these assessments involved temporally, structurally and contextually near transfer. We suggest that if mathematics learning is to be sustained and meaningful, then a far more significant problem is the assessment of far and very far transfer. Hence we designed and trialled some problems that would assess far transfer, and created a situation in which children's problem-solving attempts could be monitored and recorded.

Methodology

The study site was a metropolitan primary school with a diverse student community. The school was part of the *Make it Count* project (Thornton, Statton, & Mountzouris, 2012) and had taught mathematics through a range of elective projects that included design, art, sport and marine science. An interview process was designed, in which students were interviewed in either small groups or individually before and during their attempts to solve a mathematical problem framed in a real world context. In total four interviews were conducted with students ranging from grades 4 to 6.

After students were welcomed into the interview, some background information was collected, including the student's name, grade level, and the name of their previous teacher (as this determined the context through which they had learned mathematics). Students were then presented with one or two problems to solve, adapted from either a Maths 300 problem (Williams, 2010) or a NAPLAN test (ACARA, 2011). These problems were chosen following discussion with the teachers in the school, based on the relationship to both the formal curriculum and the activities that students had participated in during the year. Each interview lasted approximately 45 minutes. All interviews were videoed and transcribed by the researcher. The transcriptions were then read independently by two researchers to identify common themes that might enhance or diminish the effectiveness of the questions or prompts as assessments of far transfer. As this was a pilot study with a small corpus of data it was sufficient for the researchers to meet to discuss and refine identified themes rather than carrying out more formal coding.

Results

It is important to emphasise that the aim of this pilot study was to develop some principles and protocols that might be useful in selecting and designing tasks and prompts to assess far transfer of mathematical knowledge. Hence it is the tasks themselves that are the data in the study—the responses of the students are provided to illustrate the strengths and weaknesses of the tasks and prompts rather than indicating the extent to which the students could transfer knowledge.

Design principles

Contextual integrity. Rosie, Tara and Yvonne[1] worked together on the Garden Bed problem 'How many tiles are needed to make a border around different sized garden beds' (Educational Services Australia, 2010). This task requires students to identify the number of square white tiles required to completely surround a rectangular garden bed of width one tile and varying length. The students required no further prompts than "Please read the question out loud", and proceeded to confidently solve the problem for specific cases. After some prompting and discussion of patterns the students were asked about previous mathematics connections.

R: We designed the Anzac garden that we will be building.

T: We went out to the ANZAC garden and we measured it out and worked out how we could put it into the graph paper.

Researcher: Let me ask you a question. Does this activity relate to anything you've done before? In maths or numeracy?

Y: No [without hesitation!)

Y: What!! Hang on a sec... yes a tiny bit! Um... it kind of relates to a perimeter.... because this is a perimeter around here... so the green tiles are making a perimeter around the edge.

Yvonne's response that the problem reminded her of work she had done in a real garden measuring perimeter is indicative that the problem possesses what we have termed *contextual integrity*. This requires that transfer tasks be located within authentic rather than contrived situations (Beswick, 2011). However we suggest that such tasks should also include "the important attributes of real-life problem-solving, including ill-structured complex goals, an opportunity for the detection of relevant versus irrelevant information, [and] active/generative engagement in finding and defining problems as well as in solving them" (Young, 1993, p.45). In this case the students did not independently pose problems related to the more general case or search for patters that might relate to the number of green and blue tiles required.

Mathematical integrity. Christine was solving a problem from a NAPLAN test involving money: *David and Sarah both bought a T-shirt and hat and each spent the same amount of money. David's T-shirt coat $28.90 and hat cost $21.10. Sarah's T-shirt cost $30.95. How much did her hat cost?* (ACARA, 2011, Q 15)

Researcher: David spent how much?

C: $50

Researcher: Yeah! How could we work out how much Sarah's hat cost?

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[1] All names are pseudonyms.

C: By doing what we did?

Researcher: Yeah, do we add it or subtract it?

C: Subtract?

Researcher: Yeah! Can you show me how to do it?

C: I haven't learned how to do that yet!

Although with very directive prompting Christine was able to calculate the amount that David spent, this appeared to be more a procedural answer than recognition that addition was necessary. She was then unable to determine whether addition or subtraction was required to calculate the cost of Sarah's hat, and was not confident to carry out the calculation. Of course, she had been taught how to subtract, but stated herself that she had not "learned how to do that yet", and hence was unable to solve the problem.

Christine's knowledge of addition and subtraction was not deep or robust enough to enable her to solve the problem. In this case the task was therefore not a suitable one for assessing transfer, even with the very directive prompts that the researcher used to reduce her anxiety. We suggest that problems designed to measure students' capacity to transfer knowledge require *mathematical integrity*. That is they should be pitched at a level appropriate to the mathematical capacity of the students, involve significant mathematical ideas and be hard to solve without resorting to mathematics.

Cultural integrity. Judy was solving a problem relating to the number of different permutations of three flavours of ice cream that could be placed in an ice cream cone (Educational Services Australia, 2010).

Researcher: Do you like ice-cream?

J: YEAH (giggles)

Researcher: We aren't going to eat any unfortunately but the next problem is all about ice-cream! So can you tell me three of your favourite flavours?

J: Chocolate, banana and vanilla!

Researcher: Yum! Our next one is all about working out...actually you can tell me?

J: Ice-cream shop sells triple header ice-cream cones like this one in this pictures – strawberry, vanilla and banana, the triple header must have one scoop of each flavour!

J: different people arrange the scoops in different ways. For example, strawberry, then vanilla then banana. Vanilla, then banana, then strawberry!

Judy was excited about solving the problem, and was able to use materials to find all the different permutations of flavours. As indicated by Judy's enthusiastic responses the problem generated an emotional response. Furthermore she had control over the problem in that she was asked to choose her three favourite flavours. According to Beswick (2011) and Grootenboer and Zevenbergen (2007), tasks that value and build on students' cultural background and interests are essential to deep mathematical learning. We suggest that it is also an essential aspect of far transfer, and that therefore problems used to measure students' capacity to transfer knowledge must possess what we term *cultural integrity*. In some cases this cultural integrity might reflect deep connections with students' backgrounds and beliefs; in other case such as the ice cream problem it may simply be connecting with students' likes and allowing some control over the problem.

Material integrity. Rosie, Tara and Yvonne worked on the Heads and Legs problem from Maths 300 (Educational Services Australia, 2010). In this problem students were told that there were a certain number of buffalos and chickens in a farmyard and were given the number of heads and legs. They were provided with pictures of buffalos and hens to use when solving the problem.

Researcher: Okay... you may find these useful (pictures of buffalo and hens). How many legs do buffalos have?

Kids: 4 legs!

Researcher: We also have pictures of...

T: hens... they have two legs.

Researcher: What do you need to do first?

Y: put two buffalo and three hens in the yard.

Researcher: How many heads and legs do you have?

Y: 5 heads and uh ...

T: [whispers] 14 legs

While the children could not recall a like context the use of concrete materials prompted connections that enabled them to successfully work out how many buffalos and hens there were in a farmyard with eight heads and 22 legs. We suggest that the use of the pictures of buffalos and hens gave the problem *material integrity*, in that the materials were realistic, and provided both a stimulus and a resource for solving problems (Beswick, 2011).

Prompting for transfer

We began with a set of scripted prompts that we could give to each student or small group of students to help them to solve the problem. The initial intent was to only read as many prompts as necessary in order to scaffold the student sufficiently to solve the problem, and to obtain a measure of transfer from the number of prompts used. However it was clear in the course of conducting the interviews that our initial set of prompts was not sufficiently flexible to allow students to solve the problems or to describe how what they were doing related to other things they had done. Hence rather than using a pre-defined sequence of prompts some were given to demonstrate the next step or some asked open-ended questions linking back to their experiences. Thus we suggest that the number of prompts required, although it may be adequate as a measure of near transfer (Brown and Campione, 1984), is at best a crude approximation to assessing students' capacity to undertake far transfer. Rather richer data that arises only in the course of an interview is required.

In particular we suggest that two types of prompts are helpful in determining the extent to which students can transfer knowledge.

Connecting prompts.

- Do you (do X) in real life? What sort of maths do you use? Might this help to solve the problem?
- Have you seen or done something like this before? What sort of maths did you use to help you solve the problem?
- Do you remember when you learned X with teacher Y. Do you think this might help?

Reasoning prompts.

- Read the task out loud. What is the task asking you to do?
- What information is most important in the problem?
- Can you show me how to (do a related piece of mathematics)?
- Here, I will show you how to do (a related piece of mathematics). Now can you solve the problem?

The connecting and reasoning prompts are presented in ascending order of directedness. Thus a student who requires only the first connecting or reasoning prompt shows a greater capacity to transfer knowledge than a student who requires the very directed prompt that targets some specific aspect of their learning. In the transcripts above Rosie, Tara and Yvonne required only the second connecting prompts ("Let me ask you a question. Does this activity relate to anything you've done before? In maths or numeracy?") to successfully solve the Garden Beds problem. On the other hand Judy required extensive reasoning prompts ("Can you show me how to do it?") and remained unsuccessful in solving the NAPLAN money problem.

In our discussions following the interviews we identified a third type of prompt that we felt would have been helpful in assessing the extent to which students could transfer knowledge. We term these reflecting prompts, to be asked at the conclusion of the interview. A careful recording and analysis of students' responses to these prompts may be helpful in assessing transfer.

Reflecting prompts.

- Now can you think of a time when you have done something like this before?
- Could you use the way you have solved this problem to help in real life?

Discussion and Conclusion

The notion of transfer is problematic and contested. While there is widespread agreement that a key goal of mathematics education is for students to develop the capacity to solve a range of problems in both familiar and new contexts (Australian Curriculum and Assessment Reporting Authority [ACARA], 2013; Bransford & Schwartz, 1999), it is not clear how transfer of knowledge can best be enhanced or assessed in the school environment. To date most attempts to measure transfer have focused on near transfer in artificial situations. Based on trials with upper primary aged children we have proposed a set of design principles and a categorisation of associated prompts for tasks that can assess far transfer. Taken together the design principles (contextual, mathematical, cultural and material integrity) and prompt categories (connecting, reasoning and reflecting) may assist researchers to determine the extent to which students are able to transfer knowledge learned in one context or time to another.

We acknowledge that the questions and interviews were conducted with a very small number of students in one school. However this is one of the few attempts we have found that specifically aims to develop tasks that might assess far transfer. Given that using mathematics in unfamiliar contexts is an explicit goal of the school mathematics curriculum (ACARA, 2013), having tasks that effectively assess that goal is an urgent need. The design principles and prompt categorisation above may be a first step in the further development of more reliable assessment methods to assess far transfer.

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